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Heat Transfer at a Stretching/Shrinking Surface Beneath an External Uniform Shear Flow with a Convective Boundary Condition

(Pemindahan Haba Pada Permukaan Meregang/Mengecut di bawah Aliran Ricih Luar yang Seragam dengan Syarat Sempadan Berolak)

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ABSTRACT

The heat transfer behaviour of a viscous fluid over a stretching/shrinking sheet driven by a uniform shear in the far field with a convective surface boundary condition is studied. The boundary layer equations governing the flow are reduced to ordinary differential equations using a similarity transformation. Using a numerical technique, these equations are then solved to obtain the temperature distributions and the heat transfer rate at the surface for various values of Prandtl number, stretching/shrinking parameter and convective parameter. Dual solutions are found to exist for the shrinking case, whereas for the stretching case, the solution is unique.

Keywords: Convective boundary condition; dual solutions; heat transfer; shear flow; stretching/shrinking sheet

ABSTRAK

Tingkah laku pemindahan haba bagi bendalir likat di atas helaian meregang/mengecut dipandu oleh aliran ricih seragam dengan syarat sempadan permukaan berolak dikaji. Persamaan-persamaan lapisan sempadan yang menakluk aliran diturunkan kepada persamaan-persamaan pembezaan biasa menggunakan penjelmaan keserupaan. Menggunakan kaedah berangka, persamaan-persamaan tersebut kemudian diselesaikan untuk memperoleh taburan suhu dan kadar pemindahan haba pada permukaan bagi pelbagai nilai nombor Prandtl, parameter meregang/mengecut dan parameter olakan. Penyelesaian dual didapati wujud bagi kes mengecut, manakala bagi kes meregang, penyelesaian adalah unik.

Kata kunci: Aliran ricih; helaian meregang/mengecut; pemindahan haba; penyelesaian dual; syarat sempadan berolak

Introduction

The behaviour of boundary layer flow and heat transfer over a stretching sheet has been comprehensively studied until recently. Since the pioneering study by Crane (1970) on a viscous fluid over a linearly stretching plate, many aspects of this problem have been investigated by many authors. The flow with slip effect at the boundary for instance has been studied by Abbas et al. (2009), Andersson (2002), Fang et al. (2009) and Wang (2002). On the other hand, similar problems but without the slip effects have been considered by Dutta and Roy (1985), Elbashbeshy (1998), Ishak et al. (2006, 2007, 2009), Lin and Chen (1998), and Mahapatra and Gupta (2001) among others.

The flow due to a shrinking sheet has attracted a considerable interest of many researchers recently due to its different behaviours in the flow dynamics compared to the stretching case. As mentioned by Miklavčič and Wang (2006), Wang (2008), Ishak et al. (2010) and Bhattacharyya et al. (2011), a steady boundary layer flow induced by a shrinking sheet is not possible since the vorticity generated in this case is not restricted inside the boundary layer. There are some other external forces that are needed to confine the vorticity within the boundary layer to make the steady flow possible. The non-uniqueness of steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter was studied by Miklavčič and Wang (2006) and they have reported an exact solution of the Navier-Stokes equations. Hayat et al. (2008) investigated the MHD rotating flow of a second grade fluid over a porous shrinking surface using homotopy analysis method (HAM), while Noor and Hashim (2009) studied the MHD flow and heat transfer due to a shrinking sheet embedded in a fluid saturated porous medium. Fang et al. (2010) have solved the problem of the slip flow over a permeable shrinking surface using a second order slip flow model, where they presented an exact solution of the governing Navier-Stokes equations. Bachok et al. (2010) studied the unsteady three-dimensional boundary layer flow due to a permeable shrinking sheet, and found the existence of dual solutions in a certain range of the mass suction and the unsteadiness parameters. Very recently, Bhattacharyya and Layek (2011) investigated the effects of suction/blowing and thermal radiation on the steady boundary layer stagnation-point flow over a porous shrinking sheet.

The study of uniform shear driven boundary layer flow is seen to have fewer contributors in fluid mechanics. One of the early researches was contributed by Weidman

and Kubitschek (1997) where they reported a similarity solution of the boundary layer flow over a flat impermeable plate with free-shear flows driven by rotational velocities $U(y) = \beta y^{\alpha}$ with $\alpha > -2/3$. Later, Magyari et al. (2003) extended this problem to a permeable flat plate by taking $\alpha = -2/3$ and $\alpha = -1/2$. The heat transfer characteristics over an impermeable flat plate in outer shear flow with exponent $\alpha = -1/2$, and with an adiabatic wall has been considered by Magyari et al. (2004). Meanwhile, Cossali (2006) reported the similarity solutions of the energy and momentum boundary-layer equations for a power-law shear driven flow over a semi-infinite flat plate. The thermal boundary layer beneath an external uniform-shear flow was studied by Magyari and Weidman (2006).

The boundary layer flow concerning a convective surface boundary condition for the Blasius flow has been discussed by Aziz (2009), while Magyari (2011) revisited this work, and obtained an exact solution for the temperature boundary layer in a compact integral form. Bataller (2008) investigated the same problem by considering radiation effects on Blasius and Sakiadis flows. Later, the effects of suction and injection and stretching/shrinking of the similar analysis have been studied by Ishak (2010) and Yao et al. (2011), respectively. Motivated by the above mentioned investigations, in the present paper we investigate the heat transfer characteristics of a viscous and incompressible fluid over a stretching/shrinking sheet in a uniform shear flow with a convective surface boundary condition.

PROBLEM FORMULATION

Consider a two dimensional steady boundary layer shear flow over a stretching/shrinking sheet of ambient temperature T_{∞}, as shown in Figure 1. Following Magyari and Weidman (2006) it is assumed that the velocity of the stretching/shrinking sheet is $u_w(x) = U_w(x/L)^{1/3}$, while the velocity of the free stream is $u_e(y) = \beta y$, where x and y are Cartesian coordinates measured along the sheet and normal to it, respectively, L is the reference length, U_w is the reference stretching/shrinking velocity of the sheet and β is a constant strain rate. Under the boundary layer approximations, the basic equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components along the x and y axes, respectively, T is the fluid temperature, α is the thermal diffusivity and ν is the kinematic viscosity. The velocity boundary conditions can be expressed as:

$$v = 0,\ u = u_w(x) = U_w\left(\frac{x}{L}\right)^{1/3} \text{ at } y = 0, \tag{4}$$

$$u = u_e(y) = \beta y \quad \text{as} \quad y \to \infty.$$

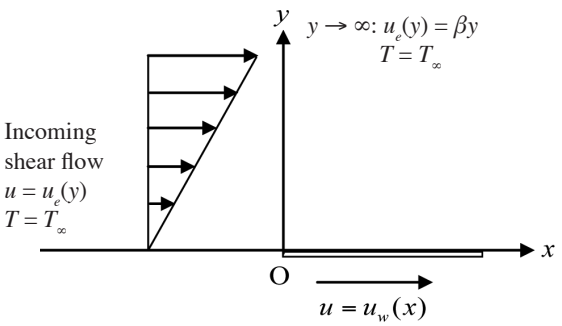


FIGURE 1. Physical model and coordinate system

Following Aziz (2009) and Ishak (2010), we assume the sheet surface temperature is maintained by convective heat transfer at a constant value T_w. Thus, the temperature boundary conditions are:

$$\begin{aligned} &-k\left(\frac{\partial T}{\partial y}\right)_{y=0} = h_f\left(T_f - T_w\right), \\ &T = T_{\infty} \quad \text{as} \quad y \to \infty \end{aligned} \tag{5}$$

where k is the thermal conductivity of the fluid, h_f is the convective heat transfer coefficient and T_f is the convective fluid temperature below the moving sheet.

Following Magyari and Weidman (2006) and Aziz (2009), we look for a similarity solution of Eqs. (1) - (3) of the following form:

$$\psi = \nu\left(\frac{x}{L}\right)^{2/3} f(\eta), \theta(\eta) = \frac{T - T_{\infty}}{T_f - T_{\infty}}, \eta = \left(\frac{x}{L}\right)^{-1/3}\frac{y}{L}, \tag{6}$$

where ψ is the stream function, which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. A simple analysis shows that $L = (\nu/\beta)^{1/2}$. Substituting (6) into Eqs. (2) and (3), we obtain the following ordinary differential equations:

$$3f''' + 2ff'' - f'^2 = 0, \tag{7}$$

$$\frac{3}{\mathrm{Pr}}\theta'' + 2f\theta' = 0, \tag{8}$$

subject to the boundary conditions:

$$\begin{aligned} &f(0) = 0,\ \ f'(0) = \lambda,\ \ \theta'(0) = -\gamma\left[1 - \theta(0)\right], \\ &f'(\eta) \to \eta,\ \ \theta(\eta) \to 0 \ \ \text{as} \ \ \eta \to \infty. \end{aligned} \tag{9}$$

Here primes denote differentiation with respect to η, $\lambda = U_w/(\beta\nu)^{1/2}$ is the stretching/shrinking parameter and γ is given by:

$$\gamma = \frac{h_f L}{k}\left(\frac{x}{L}\right)^{1/3}. \tag{10}$$

For the thermal equation (8) to have a similarity solution, the quantity γ must be a constant and not a function of x as in Eq. (10). This condition can be met if the heat transfer coefficient h_f is proportional to $(x/L)^{-1/3}$. We, therefore assume:

$$h_f = c(x/L)^{-1/3} \tag{11}$$

where c is a constant. Thus, we have:

$$\gamma = cL/k, \tag{12}$$

with γ defined by Eq. (12), the solutions of Eqs. (7) - (9) yield the similarity solutions. However, with γ defined by Eq. (10), the generated solutions are local similarity solutions. It should be mentioned that the constant temperature results are recovered by using a value of $\gamma = \infty$ in the boundary conditions (9), which then gives the condition $\theta(0) = 1$ (isothermal condition) and Eqs. (7)-(9) are identical with Eqs. (7) and (8) from the paper by Magyari and Weidman (2006) when $\lambda = 0$ and $m = 0$.

The quantities of physical interest are the skin friction coefficient and the local Nusselt number which are proportional to $f''(0)$ and $-\theta'(0)$, respectively. Thus, our task is to investigate how these quantities vary with the governing parameters.

NUMERICAL METHOD

The nonlinear differential equations (7) and (8) along with the boundary conditions (9) form a two point boundary value problem (BVP) and are solved using a shooting method, by converting it into an initial value problem (IVP). In this method, we choose suitable finite values of $\eta \to \infty$, say η_∞, which depend on the values of the parameters used. First, the system of equations (7) and (8) is reduced to a first-order system (by introducing new variables) as follows:

$$f' = p, \quad p' = q, \quad 3q' + 2fq - p^2 = 0, \tag{13}$$

$$\theta' = r, \quad \frac{3}{\Pr} r' + 2fr = 0, \tag{14}$$

with the boundary conditions:

$$\begin{aligned} &f(0) = 0, \quad p(0) = \lambda, \quad \theta'(0) = -\gamma\left[1 - \theta(0)\right], \\ &p(\eta_\infty) = \eta_\infty, \quad \theta(\eta_\infty) = 0. \end{aligned} \tag{15}$$

Now we have a set of 'partial' initial conditions

$$f(0) = 0, p(0) = \lambda, q(0) = \alpha_1, \theta(0) = \alpha_2, \ r(0) = -\gamma[1 - \alpha_2].$$

As we notice, we do not have the values of $q(0)$ and $\theta(0)$ (and $r(0)$, i.e. α_1 and α_2). To solve Eqs. (13) and (14) as an IVP, we need the values of α_1 and α_2. By trial and error, we guess these values, and apply Runge-Kutta-Fehlberg method in Maple software, and then see if this guess matches the boundary conditions at the very end. If we don't succeed then we try and try again. Varying the initial slopes give rise to a set of profiles which suggest the trajectory of a projectile "shot" from the initial point. That initial slope is sought which results in the trajectory "hitting" the target, that is, the final value (Bailey et al. 1968).

RESULTS AND DISCUSSION

The system of equations (7) – (9) was solved for some values of Prandtl number Pr, stretching/shrinking parameter λ and convective parameter γ. Since Eqs. (7) and (8) are uncoupled, the flow field is not affected by the thermal field. Thus, the convective parameter γ and the Prandtl number Pr have no effect on the flow field. For this reason, for each values of γ and Pr, the function $f(\eta)$ and its derivatives are identical.

The variation of the skin friction coefficient $f''(0)$ with the stretching/shrinking parameter λ is presented in Figure 2, while that of the local Nusselt number $-\theta'(0)$ for some values of Pr is presented in Figure 3. These figures show that solution exists for all positive values of the stretching/shrinking parameter λ (stretching case), while for negative values of λ (shrinking case), there is a critical value λ_c, with two solution branches for $\lambda_c < \lambda < 0$, a saddle-node bifurcation $\lambda = \lambda_c$ and no solution for $\lambda < \lambda_c$.

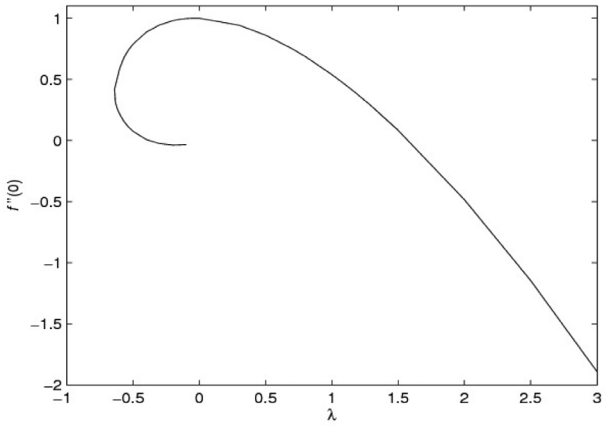


FIGURE 2. Variation of the skin friction coefficient $f''(0)$ with stretching/shrinking parameter λ

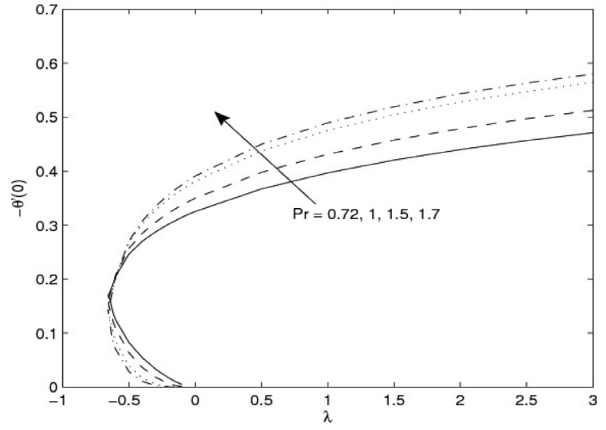


FIGURE 3. Variation of the local Nusselt number $-\theta'(0)$ with stretching/shrinking parameter λ for some values of Prandtl number Pr when $\gamma = 1$

Based on our computations, we found that λ_c = -0.6575. The selected values of $-\theta'(0)$ used to sketch Figure 3 are given in Table 1.

We identify the upper and lower branch solutions in the following discussion by how they appear in Figures 2 and 3, i.e. the upper branch solution has a higher value of $f''(0)$ for a given λ, than the lower branch solution. The effects of the Prandtl number on the temperature profiles for λ = -0.2 (shrinking case) are shown in Figure 4. It is seen that the upper branch solution yields a thinner thermal boundary layer thickness compared to the lower branch solution. The temperature profiles of the lower branch are inflated near the surface and the heat flux at the surface becomes very small. It is interesting to note how the temperature is distributed for the upper branch solution. With the increase of the Prandtl number, the temperature drops faster to the ambient temperature, thus increases the surface heat flux. Figure 5 also shows that the far field boundary conditions (9) are satisfied asymptotically, hence support the validity of the numerical results obtained, besides supporting the existence of the dual solutions shown in Figures 2 and 3.

Figure 5 illustrates the effects of the stretching rate on the temperature profiles when Pr and γ are fixed to unity. It is seen that the boundary layer thickness become thinner as λ increases, thus increase the heat transfer rate at the surface $-\theta'(0)$ with increasing values of λ. Similar behaviours are observed for the shrinking case ($\lambda < 0$) for the upper branch solution (Figure 6(a)). On the other hand, the lower branch solution exhibit an analogous pattern of temperature distributions as can be seen in Figure 6(b). Moreover, the boundary layer thickness becomes thicker as λ increases. Figure 7 shows the effects of various values of convective parameter γ when Pr and λ are fixed to unity. The temperature distributions are found to be quite similar

TABLE 1. Values of the local Nusselt number $-\theta'(0)$ for different values of stretching/shrinking parameter λ and Prandtl number Pr when the convective parameter γ = 1

λ	Pr			
	0.72	1	1.5	1.7
Stretching case				
1	0.3971	0.4315	0.4757	0.4897
1.5	0.4206	0.4577	0.5048	0.5196
2	0.4401	0.4791	0.5282	0.5435
2.5	0.4568	0.4972	0.5477	0.5633
3	0.4714	0.5129	0.5644	0.5802
Shrinking case				
-0.1	0.3145 (0.0051)	0.3372 (0.0011)	0.3661 (0.0001)	0.3751 (0)
-0.2	0.3021 (0.0170)	0.3227 (0.0068)	0.3486 (0.0012)	0.3566 (0.0005)
-0.3	0.2876 (0.0336)	0.3057 (0.0182)	0.3279 (0.0058)	0.3347 (0.0030)
-0.5	0.2466 (0.0827)	0.2571 (0.0635)	0.2680 (0.0392)	0.2706 (0.0288)
-0.6	0.2051 (0.1247)	0.2077 (0.1082)	0.2062 (0.0852)	0.2044 (0.0703)

*Results for the lower branch solution are given in parentheses.

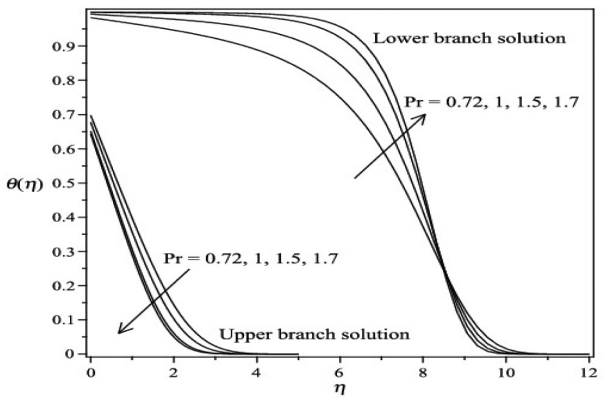


FIGURE 4. Temperature profiles $\theta(\eta)$ for some values of Pr when λ = -0.2 and γ = 1

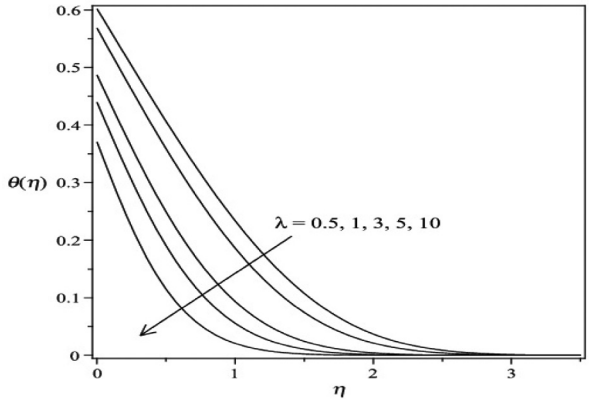
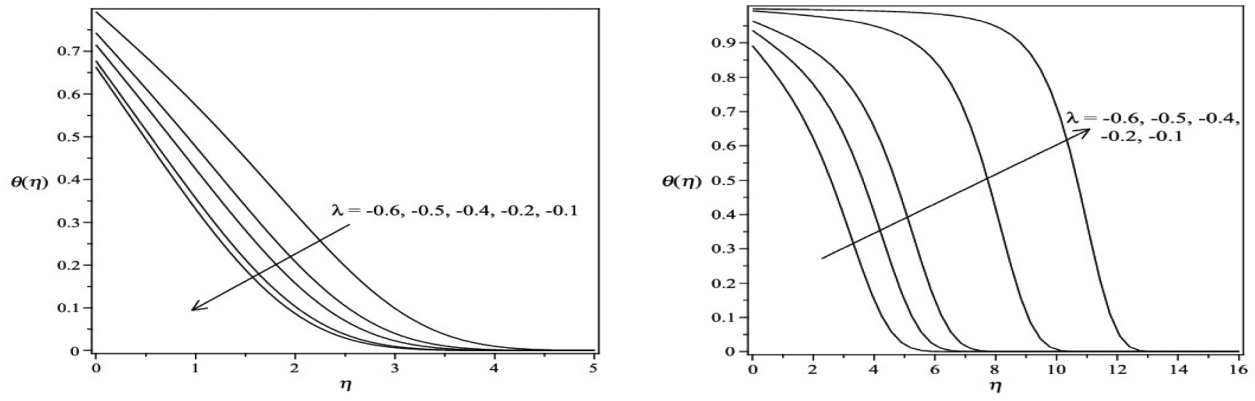


FIGURE 5. Temperature profiles $\theta(\eta)$ for some values of λ (> 0) when Pr = 1 and γ = 1



(a) (b)

FIGURE 6. Temperature profiles $\theta(\eta)$ for some values of λ (< 0) for (a) upper branch solution and (b) lower branch solution when Pr = 1 and γ = 1

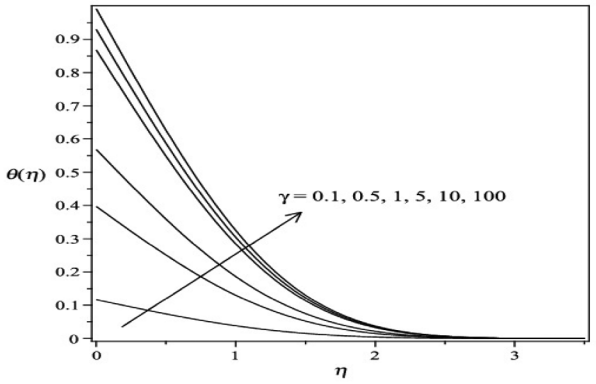


FIGURE 7. Temperature profiles $\theta(\eta)$ for some values of γ when Pr = 1 and λ = 1

with those obtained by Aziz (2009) who considered the boundary layer flow over a static flat plate ($\lambda = 0$), and by Ishak (2010) for the permeable plate case. It is clear that the surface temperature $\theta(0)$ increases as γ increases. As reported by Aziz (2009), the parameter γ at any location x is proportional to the heat transfer coefficient associated with the hot fluid h_f. The thermal resistance on the hot fluid side is inversely proportional to h_f. Therefore, the hot fluid side convection resistance decreases as γ increases and hence, the surface temperature $\theta(0)$ increases.

CONCLUSION

The problem of steady boundary layer flow and heat transfer over a stretching/shrinking sheet in the presence of an external uniform shear flow with a convective surface boundary condition was studied numerically. Similarity solution for the thermal field is possible when the convective heat transfer from the lower surface is proportional to $(x/L)^{-1/3}$, where x is the distance from the slot where the sheet is issued and L is the reference length. It was found that the heat transfer rate at the surface increases with increasing values of the convective parameter. Dual solutions were found to exist for the shrinking case, whereas for the stretching case, the solution is unique.

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